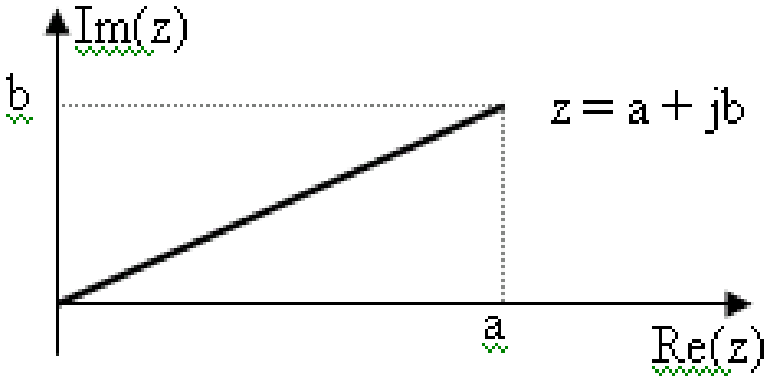
BILANGAN KOMPLEKS

Bentuk Rectangular

$z = a \pm bi$ atau $z = a \pm bj$ atau $z = a \pm jb$

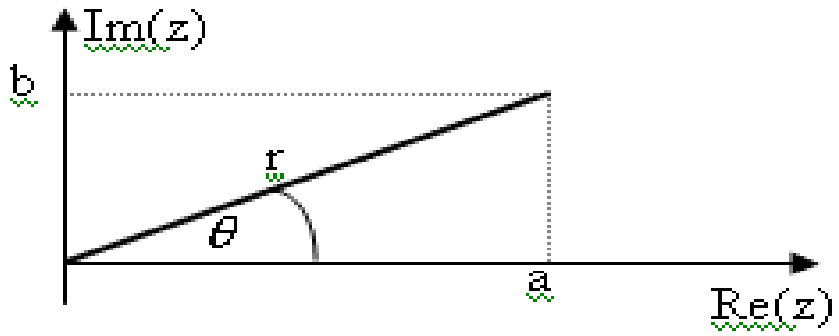
$a = \mathrm{Re}(z)$ dan $b = \mathrm{Im}(z)$



Bentuk Kutub (Polar)

$a = r \cos\theta, \quad b = r \sin\theta$

$z = r \cos\theta + j \sin\theta$



$r = \sqrt{a^2 + b^2}$, $\theta = \text{tg}^{-1}\left(\frac{b}{a}\right)$

r : modulus/amplitudo/magnitude

θ : argumen/beda fase

Bentuk Eksponensial (Euler)

$$z = re^{\pm j\theta}$$

Teorema D'Moivre

- Untuk pangkat n bulat

$$\begin{aligned} z^n &= (a + jb)^n \\ &= r^n(\cos\theta + j\sin\theta)^n \\ &= r^n(\cos n\theta + j\sin n\theta) \end{aligned}$$

Untuk pangkat n pecahan

$$Z^{1/m} = (a + jb)^{1/m}$$

$$= r^{1/m} e^{j\theta/m}$$

$$= r1/m \left(\cos \frac{\theta + 360^0 . k}{m} + j \sin \frac{\theta + 360^0 . k}{m}\right)$$

Operasi Uner

- Konjugasi

  Jika $z = a \pm jb$ maka $\bar{z} = a \mp jb$

- Minus

  Jika $z = a \pm jb$ maka $-z = -(a \pm jb) = -a \mp jb$

Operasi Biner

Jika : $z_1 = a_1 + jb_1$ dan $z_2 = a_2 + jb_2$

- Penjumlahan

$$\begin{aligned} z_1 + z_2 &= (a_1 + jb_1) + (a_2 + jb_2) \\ &= (a_1 + a_2) + j(b_1 + b_2) \end{aligned}$$

- Perkalian

$$\begin{aligned} z_1 z_2 &= (a_1 + jb_1)(a_2 + jb_2) \\ &= a_1 a_2 + j(b_1 a_2 + a_1 b_2) + j^2 b_1 b_2 \\ &= (a_1 a_2 - b_1 b_2) + j(b_1 a_2 + a_1 b_2) \end{aligned}$$

- Pembagian

$$\frac{z_1}{z_2} = \frac{a_1 + jb_1}{a_2 + jb_2} x \frac{a_2 - jb_2}{a_2 - jb_2}$$

$$= \frac{a_1a_2 + j(b_1a_2 - a_1b_2) - j^2b_1b_2}{a_2^2 - j_2b_2^2}$$

$$= \frac{(a_1a_2 + b_1b_2) + j(b_1a_2 - a_1b_2)}{a_2^2 + b_2^2}$$

Sifat-sifat Uner dan Biner

- $\overline{(z_1 + z_2)} = \overline{z_1} + \overline{z_2}$
- $\overline{(z_1 z_2)} = \overline{z_1}.\overline{z_2}$
- $=$
- $\overline{\overline{z}} = z$
- $z.\overline{z} = |z|^2 = |\overline{z}|^2 = a^2 + b^2$
- $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

Hukum-hukum yang berlaku :

- Hukum Komutatif

  Penjumlahan : $z_1 + z_2 = z_2 + z_1$

  Perkalian : $z_1 z_2 = z_2 z_1$

- Hukum Asosiatif

  Penjumlahan : $z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$

  Perkalian : $z_1(z_2 z_3) = (z_1 z_2) z_3$

- Hukum Distributif

  $(z_1 + z_2)\, z_3 = z_1 z_2 + z_2 z_3$

Aljabar Phasor (vektor yang berputar)

$$r\angle\theta$$

Sifat-sifat aljabar Phasor

1. $r_1\angle\theta \pm r_2\angle\theta = (r_1 \pm r_2)\angle\theta$

2. $r_1\angle\theta_1 . r_2\angle\theta_2 = (r_1 . r_2)\angle(\theta_1 + \theta_2)$

3. $\dfrac{r_1\angle\theta_1}{r_2\angle\theta_2} = \dfrac{r_1}{r_2}\angle(\theta_1 - \theta_2)$

SOAL-SOAL

1. Jika $z_1 = 20\angle -45^\circ$, $z_2 = 30\angle 30^\circ$, $z_3 = 40\angle 60^\circ$, nyatakan dalam bentuk rectagular, polar dan phasor untuk :

   a. $z_1 - z_2$ b. $(z_2 - z_1)z_3$ c. $(z_1 + z_2)(\overline{z_2} + z_3)$

   d. $\dfrac{(z_1 + z_3)z_2}{(z_3 + z_2)\overline{z_1}}$ e. $\dfrac{(z_1 + \overline{z_3})(\overline{z_2} + z_3)}{\overline{(z_1 z_3 + z_2 z_3)}}$

   f. $\dfrac{\overline{(z_1 . z_2)}(z_2 + z_3)}{\overline{z_1} + z_2(\overline{z_3 z_1 z_2})} + z_1(\overline{z_2 - z_3})$.