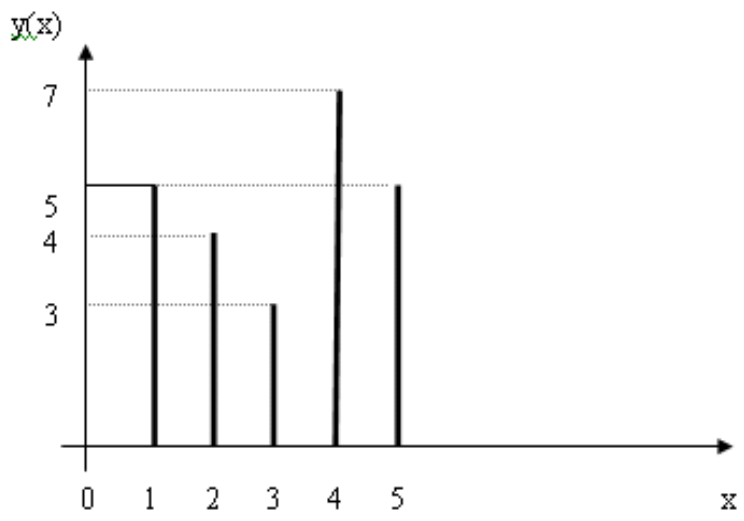
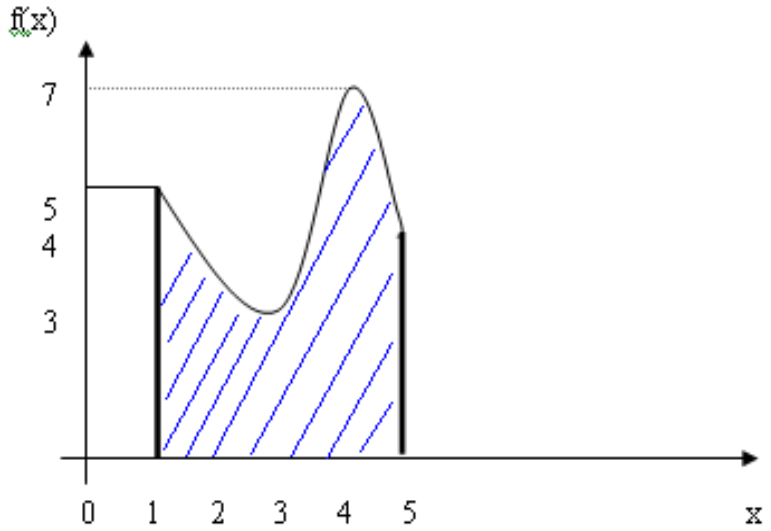
INTEGRAL

Ira Prasetyaningrum

Apa beda sigma & integral?





Nilai rata-rata gelombang fungsi y(x) adalah :

$$\text{Rata2} = \frac{\sum_i x_i}{5} = \frac{(5+4+3+7+5)}{5} = 4{,}8$$

Nilai rata-rata gelombang fungsi f(x) adalah : $\dfrac{\int_1^5 f(x)dx}{5-1}$

Integral Baku

1. $\int x^n dx = \frac{1}{n+1} x^{n+1} + c$
2. $\int \frac{1}{x} dx = \ln x + c$
3. $\int e^x dx = e^x + c$
4. $\int e^{kx} dx = \frac{1}{k} e^{kx} + c$
5. $\int a^x dx = \frac{a^x}{\ln a} + c$
6. $\int \sin x dx = -\cos x + c$
7. $\int \cos x dx = \sin x + c$
8. $\int \sec^2 x dx = \tan x + c$
9. $\int \sinh x dx = \cosh x + c$
10. $\int \cosh x dx = \sinh x + c$
11. $\int \frac{1}{\sqrt{(1-x^2)}} dx = \sin^{-1} x + c$
12. $\int \frac{-1}{\sqrt{(1-x^2)}} = \cos^{-1} x + c$
13. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$
14. $\int \frac{1}{\sqrt{(x^2+1)}} dx = \sinh^{-1} x + c$
15. $\int \frac{1}{\sqrt{(x^2-1)}} dx = \cosh^{-1} x + c$
16. $\int \frac{1}{1-x^2} dx = \tanh^{-1} x + c$

Contoh

$$\int e^{5x}\,dx = \frac{1}{5}e^{5x} + c$$

$$\int 2\sinh x\,dx = 2\cosh x + c$$

$$\int 4x^{6}\,dx = \frac{4}{7}x^{7} + c$$

$$\int \frac{5}{x}\,dx = 5\ln x + c$$

$$\int 5^{x}\,dx = \frac{5^{x}}{\ln 5} + c$$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{2}{3}x^{\frac{3}{2}} + c$$

Fungsi Suatu Fungsi Linier

Contoh : $\int (5x-4)^{7}\,dx$

Misal : $z = 5x-4 \Rightarrow \frac{dz}{dx} = 5 \Rightarrow \frac{dx}{dz} = \frac{1}{5}$

$$\therefore \int z^{7}\,dx = \int z^{7}\,\frac{dx}{dz}\,dz = \int z^{7}\left(\frac{1}{5}\right)dz = \frac{1}{5}\int z^{7}\,dz = \frac{1}{5}\frac{1}{7}z^{7} + c = \frac{1}{35}z^{7} + c$$

Soal-soal :

1. $\int \cos(2x+5)\,dx$
2. $\int \frac{1}{2x+3}\,dx$
3. $\int (2x-7)^{3}\,dx$
4. $\int e^{3x+4}\,dx$
5. $\int \frac{1}{1+(2x)^{2}}\,dx$
6. $\int \cosh(1+4x)\,dx$

Integral dalam bentuk
f′(x)/f(x) dan f(x)f′(x)

- Tinjaulah : $\int \frac{2x+3}{x^2+3x-5}dx$

Misalkan : $z = x^2 + 3x - 5 \rightarrow \frac{dz}{dx} = 2x + 3 \rightarrow dz = (2x+3)dx$

$$\therefore \int \frac{2x+3}{x^2+3x-5}dx = \int \frac{dz}{z} = \ln z + c = \ln(x^2+3x-5) + c$$

$$\therefore \int \frac{f'(x)}{f(x)}dx = \ln\{f(x)\} + c$$

- Tinjaulah : $\int \tan x \sec^2 x dx$

Misalkan : $z = \tan x \rightarrow dz = \sec^2 x dx$

$$\therefore \int \tan x \sec^2 x dx = \int z dz = \frac{1}{2}z^2 + c = \frac{1}{2}\tan^2 x + c$$

Integral Parsial

Perkalian suatu fungsi yang masing-masing fungsinya
bukan koefisien diferensial dari yang lain

Contoh : $\int x^2 \ln x dx$ $\rightarrow$ ln x bukan koefisien diferensial dari x^2 dan

x^2bukan koefisien diferensial dari ln x

Jika u dan v adalah fungsi x, maka :

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx} \rightarrow uv = \int u\frac{dv}{dx}dx + \int v\frac{du}{dx}dx$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx \rightarrow \int udv = uv - \int vdu$$

Contoh

1. $\int x^2 \ln x dx$

Misal : $u = \ln x \rightarrow du = \frac{1}{x} dx$

$dv = x^2 dx \rightarrow v = \frac{1}{3} x^3$

$\therefore \int x^2 \ln x dx = \ln x(\frac{x^3}{3}) - \frac{1}{3}\int x^3 \left(\frac{1}{x}\right) dx = \frac{x^3}{3} \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{x^3}{3} \ln x - \frac{1}{3}\left(\frac{1}{3} x^3\right) + c = \frac{x^3}{3}\left(\ln x - \frac{1}{3}\right) + c$

2. $\int x^2 e^{3x} dx$

Misal : $u = x^2$ dan $dv = e^{3x}$

Maka : $\int x^2 e^{3x} dx = x^2 \left(\frac{e^{3x}}{3}\right) - \frac{2}{3}\int e^{3x} x dx = \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left\{x\left(\frac{e^{3x}}{3}\right) - \frac{1}{3}\int e^{3x} dx\right\}$

$= \frac{x^2 e^{3x}}{3} - \frac{2xe^{3x}}{9} + \frac{2}{9}\frac{e^{3x}}{3} + c = \frac{e^{3x}}{3}\left\{x^2 - \frac{2x}{3} + \frac{2}{9}\right\} + c$

Soal-soal

1. $\int x^4 \cos 2x dx$
2. $\int x^4 e^{3x} dx$
3. $\int x^3 \ln(x+4) dx$
4. $\int e^{2x} \cos 4x dx$
5. $\int e^{5x} \sin 3x dx$
6. $\int x \ln x dx$
7. $\int e^{2x} x^3 dx$
8. $\int e^{\cos x} \sin x dx$

Untuk soal nomor 5) :

$$\int e^{5x} \sin 3x dx \quad \rightarrow \text{misal } u = e^{5x} \rightarrow du = 5e^{5x},\ dv = \sin 3x dx \rightarrow v = \frac{-\cos 3x}{3}$$

$$\therefore \int e^{5x} \sin 3x dx = e^{5x}\left(\frac{-\cos 3x}{3}\right) + \frac{5}{3}\int e^{5x} \cos 3x dx = \frac{-e^{5x} \cos 3x}{3} + \frac{5}{3}\left\{e^{5x}\left(\frac{\sin 3x}{3}\right) - \frac{5}{3}\int \sin 3x e^{5x} dx\right\}$$

$$w = \frac{-e^{5x} \cos 3x}{3} + \frac{5}{9} e^{5x} \sin 3x - \frac{25}{9} w \quad \rightarrow \frac{34}{9} w = \frac{e5x}{3}\left\{\frac{5}{3}\sin 3x - \cos 3x\right\} + c1$$

$$w = \frac{3}{34} e^{5x}\left\{\frac{5}{3}\sin 3x - \cos 3x\right\} + c$$

Integral Dengan Pecahan Parsial

Misal $\int \frac{x+1}{x^2-3x+2}dx$

Bentuk tersebut tidak termasuk dalam jenis baku. Namun dapat dinyatakan :

$\frac{3}{x-2} - \frac{2}{x-1}$ sebagai bentuk pecahan parsial

Kaidah-kaidah :

i). Derajat fungsi pembilang harus lebih rendah dari fungsi penyebut.

ii). Faktorkan penyebut dengan faktor-faktor primanya.

iii). Faktor linier $(ax+b)$ pecahan parsialnya berbentuk $\frac{A}{ax+b}$

iv). Faktor $(ax+b)^2 \rightarrow \frac{A}{ax+b} + \frac{B}{(ax+b)^2}$

v). Faktor $(ax+b)^3 \rightarrow \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \frac{C}{(ax+b)^3}$

vi). Faktor kuadrat : $(ax^2+bx+c) \rightarrow \frac{Ax+B}{ax^2+bx+c}$

Contoh

1. $\int \frac{x+1}{x^2-3x+2}dx$

$$\frac{x+1}{x^2-3x+2} = \frac{x+1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$$

Kalikan kedua ruas dengan penyebut :

$$x+1 = A(x-2) + B(x-1)$$

- Ambil $(x-1) = 0 \rightarrow x = 1$
  $\therefore\ 2 = A(-1) + B(0) \rightarrow A = -2$
- Ambil $(x-2) = 0 \rightarrow x = 2$
  $\therefore\ 3 = A(0) + B(1) \rightarrow B = 3$

Sehingga dapat ditulis :

$$\int \frac{x+1}{x^2-3x+2}dx = \int \frac{3}{x-3}dx - \int \frac{2}{x-1}dx = 3\ln(x-2) - 2\ln(x-1) + c$$

Contoh

2. $\int \frac{x^2}{(x+1)(x-1)^2}dx$

$$\frac{x^2}{(x+1)(x-1)^2} = \frac{A}{(x+1)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}$$

$$x^2 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

- $x = 1 \rightarrow 1 = 0 + 0 + 2C \rightarrow C = \frac{1}{2}$
- $x = -1 \rightarrow 1 = 4A + 0 + 0 \rightarrow A = \frac{1}{4}$

Untuk mencari konstanta B dengan menyamakan koefisien.

Pilih pangkat tertinggi yaitu x^2.

$$x^2 = Ax^2 + Bx^2$$

$$1 = A + B \rightarrow B = 1 - A = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\int \frac{x^2}{(x+1)(x-1)^2}dx = \frac{1}{4}\int \frac{1}{x+1}dx + \frac{3}{4}\int \frac{1}{x-1}dx + \frac{1}{2}\int (x-1)^{-2}dx = \frac{1}{4}\ln(x+1) + \frac{3}{4}\ln(x-1) - \frac{1}{2(x-1)} + c$$

Soal-soal

1. $\int \frac{x^2+1}{(x+2)^2}\,dx$

2. $\int \frac{x^2}{(x-2)(x^2+1)}\,dx$

3. $\int \frac{x^2-1}{(x+1)(x^2-3x+1)}\,dx$

4. $\int \frac{4x^2+1}{x(2x-1)^2}\,dx$

6. $\int \frac{dx}{x^2(1+x^2)}$

7. $\int \frac{2x+3}{(x-3)(x+1)^2}\,dx$

Integral Lipat Dua

$$\iint f(x,y)\,dx\,dy = \int \left\{ \int f(x,y)\,dx \right\} dy$$

$$\iint f(x,y)\,dy\,dx = \int \left\{ \int f(x,y)\,dy \right\} dx$$

Contoh :

1. $$\iint 3x^2 y\,dx\,dy = \int \left\{ \int 3x^2 y\,dx \right\} dy = \int x^3 y\,dy = \frac{1}{2} x^3 y^2 + c$$

2. $$\iint (x^2 - 2y)^2\,dy\,dx = \int \left\{ \int (x^2 - 2y)\,dy \right\} dx = \int (x^2 y - y^2)\,dx = \frac{1}{3} x^3 y - xy^2 + c$$

Contoh Integral Tertentu