Teknik Informatika – PENS-ITS

ALJABAR LINIER

PERTEMUAN 2-4

TUJUAN INSTRUKSIONAL KHUSUS

Setelah menyelesaikan pertemuan ini mahasiswa diharapkan :

- Mengetahui definisi Sistem Persamaan Linier
- Dapat membentuk matriks yang merepresentasikan Sistem Persamaan Linier
- Dapat menyelesaikan Sistem Persamaan Linier dengan menggunakan metode Gauss dan Gauss Jordan

Contoh Soal ➔ berapa nilai x, y dan z

$x + y + 2z = 9$

$2x + 4y - 3z = 1$

$3x + 6y - 5z = 0$

Sistem Persamaan Linier

Persamaan linier :

Persamaan yang semua variabelnya berpangkat 1 atau 0 dan tidak terjadi perkalian antar variabelnya.

Contoh: **(1)** $x + y + 2z = 9$ → **PL**

(2) $2x + y = 9$ → **PL**

(3) $2xy – z = 9$ → **Bukan PL**

Solusi PL (1) : berupa suatu “tripel” dengan masing-masing nilai sesuai urutan **(nilai-x, nilai-y, nilai-z)** yang memenuhi persamaan tersebut.

Himpunan solusi untuk persamaan di atas:

{ … (0, 1, 4), (1, 0, 4), (4, 5, 0), …. }

Himpunan solusi juga disebut Ruang Solusi (*solution space*)

Misal :

$$z = t \rightarrow 0$$

$$y = s \rightarrow 5$$

$$x = 9 - s - 2t \rightarrow 4$$

atau

$$x = t \rightarrow 4$$

$$y = s \rightarrow 5$$

$$z = \frac{9 - t - s}{2} \rightarrow 0$$

atau

$$x = t \rightarrow 4$$

$$z = s \rightarrow 0$$

$$y = 9 - 2s - t \rightarrow 5$$

- terserah variable mana yang akan diumpamakan, rumus berbeda, tapi hasil akhir untuk x, y, dan z tetap sama

Sistem Persamaan Linier:

Suatu sistem dengan beberapa (2 atau lebih) persamaan linier.

Contoh:

$$\begin{aligned} x + y &= 3 \\ 3x - 5y &= 1 \end{aligned}$$

Ruang Solusi:

berupa semua ***ordered-pair*** **(nilai-x, nilai-y)** yang harus memenuhi semua persamaan linier dalam sistem tersebut;

untuk sistem ini ruang solusinya { **(2, 1)** }

PENYIMPANGAN PADA PENYELESAIAN SUATU SPL

Pada beberapa SPL tertentu terdapat penyimpangan – penyimpangan dalam penyelesaiannya, misal :

Diberikan SPL sebagai berikut :

$$x_1 + 1/2x_2 + 1/3x_3 = 1$$
$$1/2x_1 + 1/3x_2 + 1/4x_3 = 0$$
$$1/3x_1 + 1/4x_2 + 1/5x_3 = 0$$

Didapat penyelesaian $x_1 = 9$, $x_2 = -36$, dan $x_3 = 30$

Jika SPL tersebut dituliskan dalam bentuk dua desimal :

$$x_1 + 0{,}5x_2 + 0{,}33x_3 = 1$$
$$0{,}5x_1 + 0{,}33x_2 + 0{,}25x_3 = 0$$
$$0{,}33x_1 + 0{,}25x_2 + 0{,}2x_3 = 0$$

Didapat penyelesaian $x_1 \approx 55{,}55$; $x_2 \approx -277{,}778$; dan $x_3 \approx 255{,}556$

Interpretasi Geometrik:

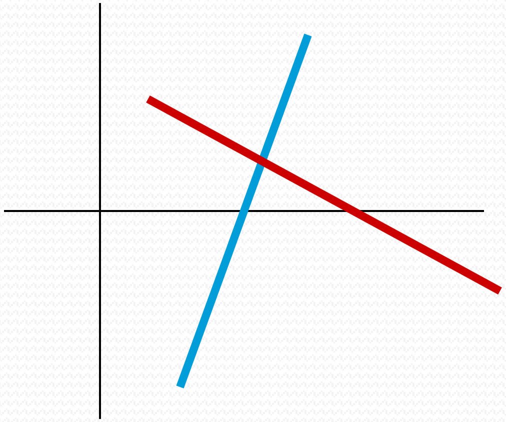
Sistem menggambarkan 2 garis lurus pada sebuah bidang datar.

g_1: $\mathbf{x + y = 3}$

g_2: $\mathbf{3x - 5y = 1}$

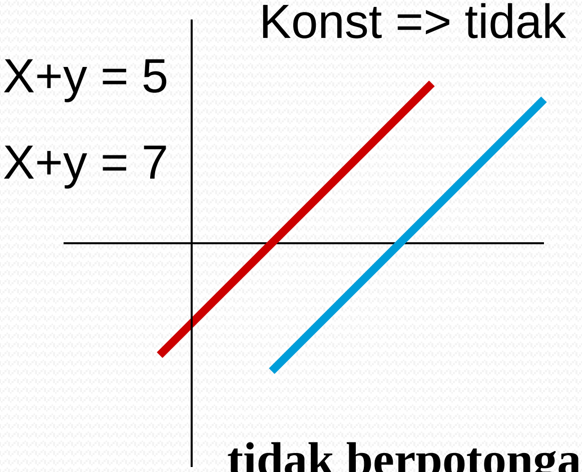
Solusi: g_1 dan g_2 berpotongan di **(2, 1)**

Kemungkinan:



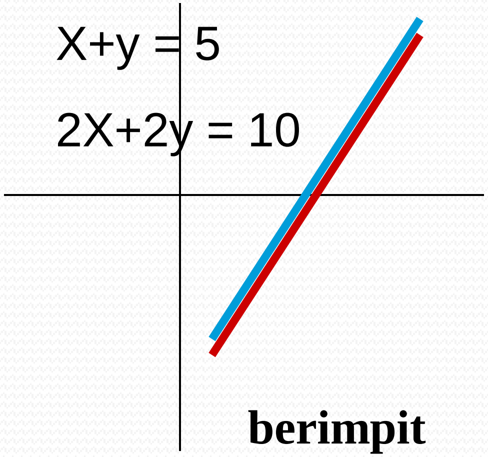
berpotongan di 1 titik

Var => sama
Konst => tidak



tidak berpotongan

Kelipatan



berimpit

Solusi Sistem Persamaan Linier

a. Cara Biasa → Seperti SMA

b. Eliminasi Gauss

c. Eliminasi Gauss - Jordan

a. Cara Biasa (untuk mengingat kembali):

I.

$$\begin{array}{rcl} x + y = 3 & \rightarrow & 3x + 3y = 9 \\ 3x - 5y = 1 & \rightarrow & \underline{3x - 5y = 1} \quad - \\ & & 8y = 8 \quad \rightarrow \quad y = 1 \end{array}$$

$$3x - 5 = 1 \rightarrow 3x = 6 \rightarrow x = 2$$

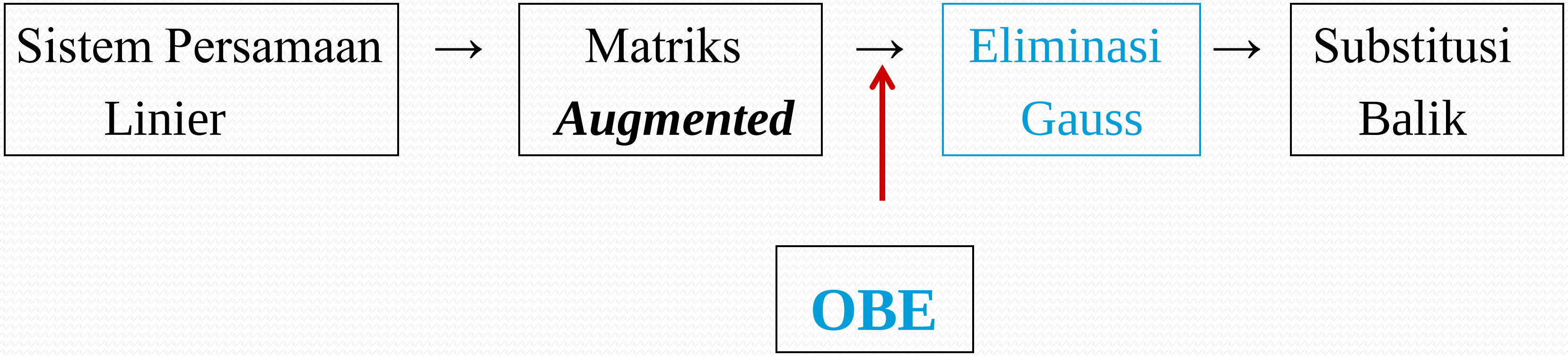
II.

$$y = 3 - x$$

$$3x - 5(3 - x) = 1 \text{ atau } 3x - 15 + 5x = 1 \rightarrow 8x = 16 \rightarrow x = 2$$

$$y = 3 - x \rightarrow y = 1$$

b. Eliminasi Gauss (ringkasan):



Sistem Persamaan Linier → Matriks ***Augmented*** → Eliminasi Gauss → Substitusi Balik

OBE

Penyelesaian Sistem Persamaan Linier

b. Eliminasi Gauss

$$\left.\begin{aligned} x + y + 2z &= 9\\ 2x + 4y - 3z &= 1\\ 3x + 6y - 5z &= 0\end{aligned}\right\} \xrightarrow[\text{bentuk matriks \textit{augmented}}]{\text{ditulis dalam}} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9\\ 2 & 4 & -3 & 1\\ 3 & 6 & -5 & 0\end{array}\right)$$

lalu diusahakan berbentuk

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9\\ 0 & 1 & ? & ?\\ 0 & 0 & 1 & ?\end{array}\right)$$

dengan proses **Operasi Baris Elementer (OBE)**

(Elementary Row Operation - ERO)

Matriks *Augmented* : (Matriks yang diperbesar)

Matriks yang entri-entrinya dibentuk dari koefisien-koefisien Sistem Persamaan Linier

Contoh :

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

Matriks *Augmented*-nya :

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right)$$

Operasi Baris Elementer (OBE)

(Elementary Row Operation - ERO)

Perhatikan bahwa tiap baris dari matriks merepresentasikan persamaan linier

1. **Mengalikan suatu baris dengan bilangan nyata k $\neq$ 0**
2. **Menukar posisi dua baris**
3. **Menambah baris-i dengan k kali baris-j**

Ciri-ciri eliminasi Gauss (Eselon Baris) :

1. Entri-entri dalam sebuah baris tidak semuanya nol, maka entri pertama yang tidak nol harus 1 (disebut 1-utama / *leading-1*)
2. Baris-baris yang semua entrinya 0, dikelompokkan di bagian bawah matriks
3. Posisi 1-utama dari baris yang lebih bawah harus lebih ke kanan d/p 1-utama baris yang lebih atas

CONTOH :

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 2 & 6 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Ciri-ciri eliminasi Gauss Jordan (Eselon Baris Tereduksi):

1, 2, 3, ditambah

4. Semua entri (yang lain) dari kolom yang berisi 1-utama harus di-0-kan

CONTOH :

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Eliminasi Gauss menggunakan O.B.E :

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{bmatrix}$$

[baris 1 -2] + baris 2

$$\begin{pmatrix} 1 \\ 1 \\ 2 \\ 9 \end{pmatrix} * \begin{pmatrix} -2 \\ -2 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ -7 \\ -17 \end{pmatrix}$$

[baris 1 -3] + baris 3

$$\begin{pmatrix} 1 \\ 1 \\ 2 \\ 9 \end{pmatrix} * \begin{pmatrix} -3 \\ -3 \\ -3 \\ -3 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \\ -5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -11 \\ -27 \end{pmatrix}$$

baris 2 * 1/2

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & -1/2 & -3/2 \end{bmatrix}$$

[baris 2 -3] + baris 3

$$\begin{pmatrix} 0 \\ 1 \\ -7/2 \\ -17/2 \end{pmatrix} * \begin{pmatrix} -3 \\ -3 \\ -3 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \\ -11 \\ -27 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1/2 \\ -3/2 \end{pmatrix}$$

baris 3 -2

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Substitusi Balik

$$\boxed{z = 3}$$

$$y - \frac{7}{2} z = -17/2$$

$$y - 7/2(3) = -17/2$$

$$y = 2$$

$$x + y + 2z = 9$$

$$x + 2 + 2(3) = 9$$

$$x = 1$$

$$\therefore\ x = 1,\ y = 2,\ z = 3$$

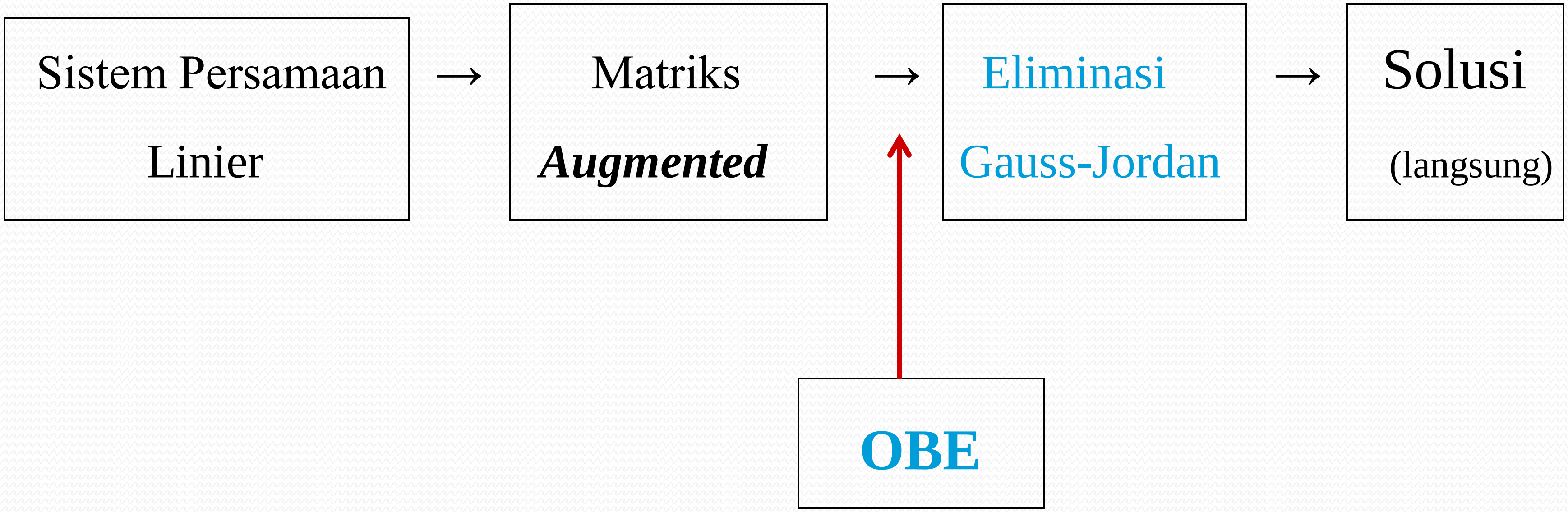
Substitusi Balik:

$$\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \end{array} \longrightarrow \quad -{}^{1}/_{2}\, z = -{}^{3}/_{2} \longrightarrow z = 3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \; z \longrightarrow \quad \begin{array}{l} 2y - 7z = -17 \\ 2y = 21 - 17 \longrightarrow y = 2 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right) \; \begin{array}{l} y \\ z \end{array} \longrightarrow \quad \begin{array}{l} x + y + 2z = 9 \\ x = -2 - 6 + 9 \longrightarrow x = 1 \end{array}$$

c. Eliminasi Gauss-Jordan (ringkasan):



Sistem Persamaan Linier → Matriks *Augmented* → Eliminasi Gauss-Jordan → Solusi (langsung)

OBE

Eliminasi Gauss-Jordan (contoh yang sama)

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned} \qquad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{array}\right)$$

dan diusahakan berbentuk

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & ? \\ 0 & 1 & 0 & ? \\ 0 & 0 & 1 & ? \end{array}\right)$$

dengan proses **Operasi Baris Elementer (OBE)**

(Elementary Row Operation - ERO)

Eliminasi Gauss-Jordan menggunakan O.B.E

- idem Gauss
- disambung dengan :

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{baris 3} \otimes \frac{7}{2} + \text{baris 2}$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \end{pmatrix} * \begin{pmatrix} 7/2 \\ 7/2 \\ 7/2 \\ 7/2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ -7/2 \\ -17/2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \longrightarrow \text{baris 3} \otimes \text{-2} + \text{baris 1} \longrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \end{pmatrix} * \begin{pmatrix} -2 \\ -2 \\ -2 \\ -2 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 2 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \longrightarrow \text{baris 2} \otimes \text{-1} + \text{baris 3} \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 2 \end{pmatrix} * \begin{pmatrix} -1 \\ -1 \\ -1 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \longrightarrow \begin{aligned} x &= 1 \\ y &= 2 \\ z &= 3 \end{aligned}$$

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

Step 1. Locate the leftmost column that does not consist entirely of zeros.

$$\begin{bmatrix} 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -10 & 6 & 12 & 28 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

Leftmost nonzero column

Step 2. Interchange the top row with another row, if necessary, to bring a nonzero entry to the top of the column found in Step 1.

$$\begin{bmatrix} 2 & 4 & -10 & 6 & 12 & 28 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{bmatrix}$$

The first and second rows in the preceding matrix were interchanged

Step 3 if the entry that is now at the top of the coloumn found in step 1 is a, multiply the first row by 1/a in order to introduce a leading 1

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 2 & 4 & -5 & 6 & -5 & -1 \end{pmatrix}$$

The first row of the preceding matrix was multiplied by ½

step 4 add suitable multiples of the top row to the rows below so that all entries below the leading 1 to zeros

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{pmatrix}$$

-2 times the first row of the preceding matrix was added to the third row

step 5 Now cover the top row in the matrix and begin again with step 1 applied to the submatrix that remains. Continue in this way until the entire matrix is in row-echelon form

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & -2 & 0 & 7 & 12 \\ 0 & 0 & 5 & 0 & -17 & -29 \end{pmatrix}$$

left most nonzero column in the submatrix

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -3{,}5 & -6 \\ 0 & 0 & 5 & 0 & -17 & 29 \end{pmatrix}$$

The first row in the submatrix
was multiplied
by -1/2 to introduce a leading 1

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -3{,}5 & -6 \\ 0 & 0 & 5 & 0 & -17 & 29 \end{pmatrix}$$

-5 times the first row of the submatirx
was added to the second row of the submatrix
to introduce a zero below the leading 1

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -3{,}5 & -6 \\ 0 & 0 & 0 & 0 & 0.5 & 1 \end{pmatrix}$$

The top row in the submatrix was
covered, and we returned again
to the step 1

leftmost non zero column in the new submatrix

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & -3{,}5 & -6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

The first(and only) row in the submatrix
was multiplied by 2 to introduce a leading 1

The entire matrix is now in **row-echelon** form.
To find the **reduce row-echelon** form we need the following additional step

Step 6 Begining with the last nonzero row and working upward, add suitable multiplies of each row to the rows above to introduce zeros above the leading 1's

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 6 & 14 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

7/2 times the third row of the preceding matrix was added to the second row

$$\begin{pmatrix} 1 & 2 & -5 & 3 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

-6 times the third row was added
to the first row

$$\begin{pmatrix} 1 & 2 & 0 & 3 & 0 & 7 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$

5 times the second row was added to the first row

The last matrix is in **reduced row echelon** form

Sistem Persamaan Linier Homogen :

1. **Sistem Persamaan Linier dikatakan homogen jika semua suku di kanan tanda “=“ adalah 0.**
2. **Solusi Sistem Persamaan Linier Homogen:**

   Solusi Trivial (semua $x_i = 0; i = 1 .. n$): pasti ada

   Solusi Non-trivial (solusi trivial, *plus* solusi di mana ada $x_i \neq 0$)

Contoh: **lihat contoh 6 halaman 18 dan verifikasi proses penyelesaiannya**

$$
\begin{aligned}
2x_1 + 2x_2 - x_3 \qquad + x_5 &= 0 \\
- x_1 - x_2 + 2x_3 - 4x_4 + x_5 &= 0 \\
x_1 + x_2 - 2x_3 \qquad - x_5 &= 0 \\
x_3 + x_4 + x_5 &= 0
\end{aligned}
$$

Matrix Augmented –nya :

$$
\left(\begin{array}{ccccc:c}
2 & 2 & -1 & 0 & 1 & 0 \\
-1 & -1 & 2 & -3 & 1 & 0 \\
1 & 1 & -2 & 0 & -1 & 0 \\
0 & 0 & 1 & 1 & 1 & 0
\end{array}\right)
$$

Contoh: lihat contoh 6 halaman 18 dan verifikasi proses penyelesaiannya

$$\left(\begin{array}{ccccc|c} 2 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

Brs-1 × (1/2)

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

Brs-2 + brs-1

Brs-3 – brs-1

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/2 & -3 & 3/2 & 0 \\ 0 & 0 & -3/2 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/2 & -3 & 3/2 & 0 \\ 0 & 0 & -3/2 & 0 & -3/2 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

Brs-2 × (2/3)

Brs-3 × (– 2/3)

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right)$$

Brs-3 – brs-2

Brs-4 – brs-2

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \end{array}\right)$$

Brs-3 × (1/2)

Brs-4 × (1/3)

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right)$$

Brs-4 – brs-3

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc|c} 1 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

baris-1 + (1/2) × baris-2

$$\left(\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccccc:c} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 + x_2 \qquad + x_5 = 0$$

$$x_3 \qquad + x_5 = 0$$

$$x_4 \qquad = 0$$

$$x_5 = s \rightarrow x_3 + x_5 = 0 \rightarrow x_3 = - x_5$$

$$x_2 = t \rightarrow x_1 + x_2 + x_5 = 0 \rightarrow x_1 = - x_2 - x_5$$

Ruang solusinya = { (-t-s, t, -s, 0, s) }

Teorema:

Sistem Persamaan Linier Homogen dengan **variabel** lebih banyak daripada **persamaan** mempunyai tak berhingga banyak pemecahan.

Ditinjau dari matriksnya:

Sistem Persamaan Linier Homogen dengan **kolom** lebih banyak daripada **baris** mempunyai tak berhingga banyak pemecahan.

TUGAS 1

- Buku Elementary Linear Algebra 9th edition, exercise 1.2:
  - No. 6 a, 6c, 7 d, 10 b, 11 a, 14 b, 15b, 16b

Dikumpulkan pada **pertemuan ke-2**

DAFTAR PUSTAKA

- Elementary Linear Algebra 9th edition, Howard Anton – Chris Rorre, John Wiley & Sons Inc., 2005